

# Some Directions in the Development of Dynamic Meteorology in Russia in 2007–2010

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**Abstract**—A brief review of the results of investigations carried out by Russian scientists in the field of dynamic meteorology in 2007–2010 is presented. This review is based on the information prepared by the Commission on the Dynamic Meteorology of the National Geophysical Committee, Russian Academy of Sciences, and included in the general information report of the Section of Meteorology and Atmospheric Sciences at the XXV General Assembly of the International Union of Geodesy and Geophysics.

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The investigations conducted by Russian scientists in 2007–2010 in the field of dynamic meteorology and discussed in this review can be conditionally related to the following sections: general dynamics of the atmosphere, large-scale processes and weather forecast, mesoscale processes, turbulence in the boundary layer, and mathematical problems of climate and ecology.

## 1. GENERAL DYNAMICS OF THE ATMOSPHERE

In many problems of dynamic meteorology, the atmospheric dynamics can be described with the aid of an ensemble of vortices interacting with each other and waves of different scales in the ideal fluid approximation. The interest of researchers in wave motions is associated, first and foremost, with the fact that waves produce instability leading to a considerable reorganization of the general dynamic pattern (in particular, to the development of cyclones, typhoons, tornadoes, and sand devils). Romanova and Yakushkin [1] consider methods of investigating the evolution of wave disturbances in density-stratified shear flows of an ideal incompressible fluid. The motions under consideration can be described by Hamiltonian equations, which, being written in semi-Lagrangian variables, have an integral–differential form. This makes it possible to study both continuous and discontinuous solutions. Two dynamic systems are considered in the cited work. One of them describes gravity waves in a shear

flow, which develop in an undisturbed medium with sharp gradients of the density and flow velocity (the Kelvin–Helmholtz model is the simplest example). The other dynamic system describes shear and gravity–shear waves in a two-dimensional flow with sharp vorticity gradients. The cited paper presents the results of solving the problem of the dynamics of disturbances in a flow with a continuous vorticity distribution obtained when the linear wave dynamics is considered in a narrow layer with a constant gradient of undisturbed vorticity and of the linear interaction of disturbances in two layers of such a type. The approach used makes it possible to study such an interaction near the critical level and the formation of structures of the “cat’s eye” type in detail.

Paper [2] by Romanova investigates the resonance interaction of waves of a discrete and a continuous spectrum. The gravity–shear wave formed on a jump of density and vorticity of an undisturbed flow and the wave on a weak jump of vorticity similar to a wave of a continuous spectrum take part in this interaction. The evolution system for amplitudes of interacting waves was obtained on the basis of the three-layer model in the form of the Hamiltonian system of equations of the dynamics of disturbances. In the case of a linear approximation (weak coupling of waves), the conditions on the parameters of the problem at which an instability arises were established. It was shown that the inclusion of the cubic nonlinearity into the evolu-

tion system stabilizes disturbances when the coefficient in the corresponding expression is positive.

Some vortical formations in the atmosphere (for example, tropical cyclones, tornadoes, and sand devils, as well as horizontally oriented vortices in the planetary boundary layer) are characterized by a clearly pronounced helical structure. Kurgansky [3] proposed considering the downward helicity flux (through the upper boundary of the atmospheric boundary layer) as a measure of the intensity of atmospheric vortices. Using the general equation of helicity balance, the author of the cited publication defines this flux as a product of the cube of the maximal wind velocity by the width of the band swept by the maximal wind during the vortex motion. It is found that, for intense vortices in a stationary (mature) phase, the helicity flux controls the rate of its destruction by the forces of turbulent friction. The cited work presents the results of a comparative (by helicity flux values) analysis of dust vortices on the Earth and on Mars, as well as sand devils. It was found, in particular, that giant dust vortices on Mars correspond to Earth's tornadoes, although, due to the difference between the air densities on both planets, their dynamic action is more than 100 times weaker than that of tornadoes.

Three main ranges of atmospheric turbulence can be identified [4]: macroturbulence with horizontal scales from hundreds of kilometers to planetary ( $\approx 10^4$  km); mesoturbulence (with scales from kilometers to hundreds of kilometers), and microturbulence with scales less than a kilometer. In the entire spatial macroscale range, the atmosphere can be regarded as a quasi-two-dimensional (the ratio of the vertical scale to the horizontal one  $\varepsilon < 10^{-2}$ ) and quasi-geostrophic fluid (the Rossby–Kibel number  $Ro = U/lL \ll 1$ , where  $l$  is the Coriolis parameter,  $U$  is the characteristic velocity scale, and  $L$  is the characteristic spatial scale). However, the question as to at what scales the spatial spectrum of atmospheric macroturbulence is controlled by its quasi-two-dimensional character and at what scales it is controlled by its quasi-geostrophicity is extremely important due to the substantial difference between the mechanisms of formation of these two types of geophysical turbulence (see, for example, [5]).

An analysis of data of the airborne measurements performed at middle and high latitudes at heights from 9 to 14 km showed that one-dimensional horizontal spectra of the wind velocity and temperature have approximately the same shape in the scale range from 2.6 to  $10^4$  km [6]. The slope of spectral distributions was close to  $-5/3$  (but not  $-3$  or more, as follows from the theory of two-dimensional turbulence) in the high-frequency (mesoscale) part of the spectrum ( $\approx 10$ –500 km). However, on larger scales, these measurements were characterized by the law  $k^{-3}$ , where  $k$  is the wave number. Internal gravity waves produce anisotropic temperature inhomogeneities in the range of vertical scales from several meters to several kilome-

ters. Due to the action of the buoyancy forces, the vertical direction is distinguished in the presence of the density (in particular, temperature) stratification, whereas in the horizontal plane the field of temperature inhomogeneities on scales not exceeding 100 km can be regarded as locally isotropic [7]. Numerous experimental investigations of the vertical temperature spectra showed that, in a wide range of values of the vertical wave numbers  $k_z$ , they obey the “ $-3$ ” power law.

Gurvich and Chunchuzov [8] proposed a model of three-dimensional spectrum of temperature inhomogeneities generated by internal gravity waves in the atmosphere. According to this model, the vertical spectrum is described by an exponential distribution with the power “ $-3$ ” and the horizontal spectrum has three exponential asymptotic sites, two of which obey the power law “ $-3$ ” and, at the intermediate site, the power changes from  $-1$  to  $-3$ , depending on the rate of anisotropy decrease as the vertical dimension of temperature inhomogeneities increases. Gurvich and Kukharets [7] presented the results of experimental investigations into the spatial inclined and vertical spectra of temperature fluctuations in the stably stratified troposphere at heights of 2–8 km (flights were performed over northern regions of European Russia) and in the range of wave numbers from  $5 \times 10^{-4}$  to  $3 \times 10^{-2}$  rad/m. The quantitative estimates of parameters of these spectra indicate that large-scale (with the vertical scale more than 100 meters) temperature inhomogeneities are strongly extended along the Earth's surface (horizontal dimensions exceed the vertical ones by about a factor of 20). The anisotropy of inhomogeneities drops as their vertical dimensions decrease, attaining values 1.5–2 on scales of about 10 m and less.

Atmospheric macroturbulence can be characterized by several key scales. The first of these scales is the scale of energy injection into the system. It is commonly supposed that the main energy is supplied into the system from baroclinic instability being achieved, i.e., in the region of the most unstable wavelengths according to Lyapunov. The second important scale in the turbulence theory is the scale of wavelengths on which the energy dissipates. Strictly speaking, the energy dissipation in the atmosphere takes place on the Kolmogorov scale, which is negligibly small for large-scale turbulence. Intermediate scales should be allotted for the energy sink. In the first place, this relates to the planetary boundary layer, where the energy flux scale varies from macroscales to the Kolmogorov scale.

Ponomarev et al. [9], on the basis of the expansion in terms of the divergence exponents of the two-dimensional velocity field, obtained an approximate system of equations for describing quasi-two-dimensional viscous flows of an incompressible fluid with allowance for dissipative effects. The model of quasi-two-dimensional flow with the Rayleigh (linear) fric-

tion is a particular case of these equations. It is shown that the three-dimensional character of the flow, which manifests itself in the effective interaction of vortices with the horizontal and vertical axes, is responsible for the nonlinear character of friction. The authors of the cited work parametrized this interaction in the quasi-two-dimensional equations with nonlinear friction and compared the theoretical results with the data of laboratory experiments on the excitation of a spatially periodic flow of a fluid. In paper [10], Ponomorev et al. experimentally investigated the vortical flow excited by the magnetohydrodynamic method in a thin layer of a rotating fluid with the use of the laboratory setup developed at the IAP RAS. An analytical approach was elaborated for the interpretation of the experimental results, which made it possible to infer that the circulation in the upper plane controls the mechanism of nonlinear friction, which redistributes cyclonic and anticyclonic vortices outside the boundary layer.

Two-dimensional turbulent motions are in the class of physical processes whose investigation does not become simpler as the dimensionality of the space decreases. The energy transfer from small-scale motions to large-scale ones is a characteristic feature of two-dimensional turbulence. In addition to the Kolmogorov interval of the spectrum with the law  $k^{-5/3}$ , the spectral region obeying the power law “ $-3$ ” must exist due to the law of enstrophy conservation (in the absence of viscosity).

For a description of the properties of the two-dimensional turbulent motion of an incompressible fluid, Tseskis [11] used the Karman–Howarth equation for the correlation function  $B_{LL}(r) = \overline{u_L(x)u_L(x+r)}$ , where  $u_L$  is the projection of the velocity on the straight line connecting the points  $x$  and  $x+r$ , and the overbar means statistical averaging. In the cited work, particular attention is given to the choice of the function  $T(k)$  associated with nonlinear terms in the equations of motion and characterizing the energy flux in the  $k$  space. It is supposed that the number of intersections of this function with the axis  $k$  must coincide with the number of quadratic conservation laws. In the three-dimensional case, the quadratic integral is only one, and a single intersection of  $T$  with the wave-number axis is confirmed by experimental data. Such an approach made it possible to describe the inverse energy cascade, as well as the formation of coherent structures, when the spectrum attains the form of the  $\delta$  function. The coherent structures, which represent vertical formations restricted by closed streamlines, prevent the spreading of the admixture trapped by them, which makes it impossible to use the classical theories of turbulent diffusion.

Gledzer [12] considered the problem of the stability of the zonal axisymmetric solution of the system of quasi-geostrophic equations of atmospheric dynamics in the hydrostatic approximation with linear friction

and radiative Newtonian cooling. In this case, the external action on the atmosphere is specified by the background horizontal temperature gradient. The Rayleigh number  $Ra$  is the main parameter which controls the character of instability of the Hadley type regime. It is established that, for large values of  $Ra$  and small values of a certain parameter  $\tau_\varphi$ , which has the dimensionality of time and characterizes the relative role of the Earth’s rotation effects, this is an ordinary baroclinic convective instability, whereas for small  $Ra$  values and large times  $\tau_\varphi$ , this is a centrifugal instability.

The problem of stability of the flow of a stratified rotating fluid with constant vertical and horizontal velocity shears was investigated in [13], also within the framework of the quasi-geostrophic approximation, as one of the variants of the Eady problem [14]. The approach based on a description of the dynamics of disturbances in a coordinate system moving together with the flow was used for solving this problem. It is shown in the cited paper that the inclusion of the meridional shear of the zonal geostrophic flow velocity qualitatively changes the dynamics of Eady waves (wave solutions of the problem with the zero potential vorticity), which manifests itself in the alternation of stages of the smooth (oscillating in time) behavior with stages of the exponential (explosive) growth of a finite duration. Along with the dynamics of individual Eady waves, paper [13] investigates the process of generation of these waves by the initial disturbance specified by an individual spatial Fourier harmonic, which excites nonmodal waves with changing-in-time horizontal and vertical wave numbers and a nonzero potential vorticity.

An investigation of the dynamics of jet flows of a stratified fluid is among the fundamental problems of hydrodynamics. Druzhinin [15] studies the development of instability of such flows with the aid of direct numerical modeling. The initially cylindrical jet with the Gaussian velocity profile in a stably stratified (with a linear density profile) medium is considered. If the initial small disturbance of the velocity field with a wide spectrum is specified, the identified quasi-two-dimensional mode will grow exponentially. In this case, the spectral maximum associated with this mode is shifted toward smaller wave numbers compared with the maximum of an unstable spiral mode of a nonstratified jet. The instability increment is proportional to  $\sqrt{Ri}$ , where  $Ri = g\Delta\rho L/\rho U^2$  is the global Richardson number (here,  $g$  is the acceleration of gravity,  $\rho$  is the characteristic value of the fluid density, and  $\Delta\rho$  is the density change along the vertical on the scale  $L$ ). A vertical flow structure, consisting of different-polarity quasi-two-dimensional vortices located in the horizontal plane near the jet axis and accompanied by the radiation of internal waves, is formed in the process of instability development. At sufficiently large times, the instability growth becomes “saturated,” whereupon

velocity and density fluctuations dampen under the action of viscous forces.

The presence of inertial–gravity waves is characteristic of a rotating stratified fluid. In horizontally inhomogeneous shear flows, these waves can be trapped inside the shear layer. The intense wave activity observed in regions of atmospheric fronts and jet flows is often associated with such localized waves. Kalashnik [16] devoted his paper to the theoretical analysis of the structure of trapped symmetric disturbances, which showed that the location of the region of trapping is controlled by the vertical density stratification of the atmosphere. If (in the Northern Hemisphere) the characteristic Väisälä–Brunt frequency  $N = (-g/\rho \partial \rho / \partial z)^{1/2}$  is larger (smaller) than the inertial frequency  $l$ , the trapping takes place in the region of anticyclonic (cyclonic) velocity shear and the frequencies of trapped waves are smaller (larger) than  $l$ .

Kalashnik and Visheratin [17] considered the regime of cyclostrophic balance (between the pressure gradient and the centrifugal force), which is characteristic of the dynamics of intense atmospheric vortices, such as sand devils and tornadoes. For a description of motions in the axisymmetric vortex core, the authors of the cited paper found the class of exact self-similar solutions of the gas dynamics equations for the velocity components linearly depending on the distance to the axis and for temperature, whose dependence of the distance to the axis is quadratic. It is shown that small disturbances of the cyclostrophic balance state in the vortex core excite oscillations of thermohydrodynamic fields with a frequency proportional to the angular velocity of its rotation. Such oscillations can be considered a source of infrasound radiation from sand devils recorded during observations. However, if the initial deviations from the balance state are fairly large, the oscillations will be anharmonic and, under the conditions of the prevailing centrifugal force, they will lead to a considerable temperature drop on the vortex axis.

At present, much attention is given to the analysis of the stability of dynamic systems on the basis of the approach using the characteristic Lyapunov exponents. If parameters of the medium are random, the stochasticity in physical fields arises. Qualitative features of individual realizations are smoothed by the averaging of these fields over an ensemble of random parameters. In this case, the statistical means characterizing “global” scales of the region where stochastic processes take place often say nothing about the details of their development inside this region. However, there are physical processes which occur with a probability of unity and are called coherent [18], and such a “statistical coherence” can be considered a certain organization of a complex dynamic system with statistically stable characteristics.

In relation to stochastic dynamic systems, the Lyapunov exponents are averaged over an ensemble of

realization of random parameters. Klyatskin [19] showed that the thus obtained mean values coincide with the curve of a typical realization for the log-normal distribution of positive nonstationary-in-time characteristics of solutions for stochastic dynamic systems. Although the complete statistics contain all the information about a dynamic system, in practice it is possible to investigate only some of the simplest statistical characteristics associated with simultaneous and one-point distributions of probabilities. There are methods of statistical topography (see, for example, [20]) which make it possible to describe the main quantitative and qualitative features of the behavior of individual realizations of a system on the basis of such information about it. In work [19], the methods of statistical topography are applied to the problem of a statistical description of diffusion and clusterization of particles and the field of concentration of a passive admixture in random hydrodynamic flows.

## 2. LARGE-SCALE PROCESSES AND WEATHER FORECAST

It is known that, over the past five decades, the most important interannual and decadal variations in weather regimes at midlatitudes in the Northern Hemisphere are caused by phases of the North Atlantic Oscillation (NAO). Extremely large anomalies of the air temperature in the layer from the Earth’s surface to heights of the middle troposphere were observed in the same decades at the same latitudes. The diagnostic calculations [21] showed that these anomalies are unrelated to extreme anomalies of the heat influxes. Such anomalous phenomena are often explained by the intensification of planetary waves or by the influence of blocking anticyclones.

Kurbatkin [22] and Kurbatkin and Smirnov [23] presented the results of an analysis of the predominant modes of low-frequency atmospheric variability, the formation mechanisms of anomalies of the annual trend, and sudden anomalies of the planetary and continental scales. With this purpose, the authors of the cited works used the NCEP/NCAR reanalysis data over the period 1959–1998 and the results of an operational objective analysis of the Hydrometeorological Center of Russia over 2002–2007. Paper [22] pays particular attention to the role of interannual negative winter anomalies of the air temperature over continents in the stabilization of the annual trend of the present-day climate observed in the 1950s–1960s. The role of deformations of the North Atlantic dipole into the west–east planetary wave, which was observed in 1997 and 2007, is also shown.

Work [23] investigates the causes and mechanisms of the formation of extremely large anomalies of the tropospheric temperature as related to the North Atlantic Oscillation. The approach used in the cited work is based on the fact that temperature tropospheric anomalies of a continental scale in the annual

cycle can both intensify (in response to the direct action of heat influxes) and weaken (during the temperature advection opposite in sign to the heat influx). In the cited work, seasonal anomalies of the air temperature at the level 850 hPa ( $T_{850}$ ) in Eurasia are studied from the monthly mean data of the NCEP/NCAR reanalysis over the period 1959–1998. It is shown that the negative (positive) NAO phase in winter is favorable for retaining negative  $T_{850}$  anomalies in the east (west) of the continent at this time of the year. However, this dependence was critically violated (due to the limited influence of the NAO on  $T_{850}$ ) approximately two years before the onset of the extreme phenomenon. The cited work considers the mechanism of anomalous heat influx as a source of intensification of negative  $T_{850}$  anomalies in winter and positive ones in summer, which restricts the influence of the predominant dynamic mode on some regions of the continent. In particular, the authors of the cited work demonstrated the season-by-season “switch-off” of the anomalies from the heat inflow mechanism under the action of large NAO changes and the complete destruction of the annual cycle of anomalies.

Mechanisms of the stratosphere–troposphere interaction in the layer consisting of the upper troposphere and lower stratosphere have aroused great interest of researchers. Dynamically, this layer is defined as a layer in which isentropic surfaces do not lie completely in the stratosphere and do not intersect the Earth’s surface. In the winter period, large-scale meanders of the tropospheric jet propagate upward and are destructed at the level of the stratospheric jet, which slows down the stratospheric jet. Borovko and Krupchatnikov [24] investigated one of the aspects of the influence that the stratosphere has on the troposphere. The sensitivity of the tropospheric dynamics to variations in the intensity of the polar vortex in the stratosphere is studied with the use of the atmospheric general circulation model with a simple mechanism of heating specified in the Newtonian form. It is shown that variations in the thermal stratification in the stratosphere cause the following noticeable changes in the tropospheric circulation during the intensification of the polar vortex in the stratosphere: (1) the poleward displacement of the zonal flow jet in the lower troposphere; (2) the near-surface pressure decrease in the polar region; (3) a decrease in the vertical component of the wave activity flux from the troposphere into the stratosphere; and (4) the dynamic response of the troposphere to the intensification of the stratospheric polar vortex correlates well with the positive NAO phase. It is also shown in the cited paper that the zonally symmetric component of the lower troposphere response to polar vortex disturbances in the stratosphere can result from the action of synoptic-scale baroclinic waves even in the absence of stationary planetary waves.

The tropopause height and the thermal stratification of the troposphere are determined from the

dynamic balance between radiative processes and dynamic (baroclinic) fluxes of entropy (heat). The question as to how dynamic and radiative processes interact in order to sustain the static stability (mean vertical gradient of the potential temperature) and the mean meridional gradient of the potential temperature still remains an open question. It is shown in work [25] that changes in the temperature stratification, when the cooling in the stratosphere increases, affect the upper layer of the troposphere, where the stratification is controlled by radiative processes. In lower layers of the troposphere, where baroclinic nonstationary vortices contribute considerably to the dynamics, the local inclination of isentropic surfaces remains unchanged and agrees with the theoretical estimate based on the theory of baroclinic turbulence for the two-layer model of the atmosphere.

The quasi-biennial oscillation (QBO) of the zonal wind in the equatorial stratosphere at heights of about 16–50 km is a climatic process of global significance. This phenomenon can be described as the westerly and easterly phases of the zonal wind propagating downward and alternating with a period of about 28 months. The main mechanism of the QBO action on the atmospheric dynamics is associated with the modulation of transfer (mainly, by stationary waves) of the wave activity in the extratropical stratosphere, which can lead to sudden stratospheric warmings [26] and to the interaction of the QBO with other low-frequency processes, for example, with the El Niño phenomenon [27]. Regional relations of the QBO with processes in the tropics, in particular, with the duration of seasonal rains and the activity of hurricanes in the Atlantic, are also noted [28].

In spite of the QBO importance, at present only a few climatic models are capable of reproducing this phenomenon (see, for example, [29]). Ideally, the global model of atmospheric general circulation must reproduce the interaction of the whole spectrum of equatorial waves with the zonal wind in the stratosphere. The question arises: what requirements must satisfy a model in order to be able to reproduce the QBO? The main difficulty in solving this question is associated with the realization of a complex mechanism of the QBO formation involving a nonlinear interaction of the zonal mean flow and vertically propagating waves of various scales. This problem is investigated in the cycle of works [30–33] carried out in 2007–2010 at the Institute of Numerical Mathematics, Russian Academy of Sciences (INM RAS).

The first publication of this cycle (Kulyamin et al. [30]) considered two mechanisms of the QBO formation, one of which is associated with the breaking of short gravity waves and the other represents the interaction of long waves with a zonal flow. Such a division is important from the standpoint of the development of climatic models in which the generation of large-scale waves is reproduced explicitly, whereas the effects of gravity waves, which have a subgrid scale, are

parametrized. The QBO formation on the basis of the mechanism of interaction of planetary equatorial waves with the zonal flow in the equatorial stratosphere is investigated in the cited paper with the use of a simple low-parameter model of evolution of the zonally averaged wind velocity component [34]. Numerical experiments showed that this mechanism requires a high model resolution (with a grid size of less than 500 m along the vertical), because the critical layers, where the main interaction takes place, have a small vertical scale. Of course, this condition is necessary for the QBO reproduction in atmospheric general circulation (AGC) models as well. At the same time, the results of experiments with the low-parameter model [34] showed that this mechanism in global models is insufficient for exciting realistic QBOs and it is necessary to take into account the whole spectrum of wave motions at the equator. Since for models of a fairly coarse resolution the processes of propagation of gravity waves take place on a subgrid scale, the mechanism of their interaction was specified in work [30] through parametrization [35], whose choice was dictated by its use in the INM RAS climatic model. It turned out that the mechanism by which gravity waves break is self-sufficient for the excitation of oscillations of the zonal equatorial wind in the upper atmosphere and, at a certain choice of parameters, realistic QBOs arise.

Therefore, the main result of the cited work is a joint low-parameter model, which makes it possible to encompass the whole spectrum of equatorial waves and combines both QBO formation mechanisms (through the interaction of the mean flow with long waves and through the breaking of short gravity waves). In this case, planetary waves play the key role in establishing the QBO period and amplitude in lower layers of the atmosphere, whereas short gravity waves transfer the energy and control the QBO characteristics in upper layers of the atmosphere. This approach was used by Kulyamin et al. [31] for constructing an atmospheric general circulation model, which reproduces realistic QBOs of the zonal wind in the equatorial stratosphere. With this purpose, the model developed at the INM RAS was taken as a basis. This model was characterized by the horizontal resolution  $2^\circ$  in latitude and  $2.5^\circ$  in longitude and a rather coarse resolution of 39 levels along the vertical. At the standard parameter values, this model does not reproduce the QBOs in the equatorial stratosphere; however, it reproduces the semiannual oscillations (SAOs) in the upper stratosphere and mesosphere. A modification of this model through increasing the number of levels to 80 and the choice of the vertical grid size in the stratosphere equal to about 0.5 km made it possible in numerical experiments to successfully reproduce the QBOs and SAOs with characteristics close to observed ones.

The problem of formation of the QBO period, its stability, and relations to the semiannual and annual harmonics was investigated by Kulyamin and Dym-

nilov [32] and Dymnikov and Kulyamin [33]. With this purpose, the authors of the cited works used the observational data of NCEP/NCAR and ERA40 reanalyses, as well as the results of numerical experiments with the INM RAS atmospheric general circulation model (INM RAS AGCM). Analytical estimates and results of numerical experiments with low-parameter models showed a strong synchronization with SAO multiple periods in the upper atmosphere (in the transition from QBOs to SAOs region) and a weak synchronization in lower layers of the QBO spreading region. The possibility of synchronization with SAOs or the annual cycle is achieved both for the mechanism of absorption of long waves by the mean flow and in the process of short gravity waves breaking. These results make it possible to consider QBOs, SAOs, and the annual cycle as a unified system of oscillations and circulation of the upper equatorial atmosphere. The INM RAS AGC model successfully reproduced the main spectral characteristics of QBOs and SAOs, as well as specific features observed in the QBO period variability. The parametrization of the effects of internal gravity waves and vertical diffusion associated with them plays a substantial role, because QBOs with periods from 12 to 36 months can arise if the level of this diffusion varies.

It is known that the large-scale atmospheric circulation over the Siberian region has some characteristics which are caused by the geographic position (length of the continent and the influence of the Arctic Ocean), as well as specific features of the underlying surface and topography. The atmospheric circulation over Siberia was investigated in works [36, 37], which consider the dynamics of cyclones and anticyclones over Western Siberia in the period 1976–2006. For this purpose, the authors of the cited works used ground-based synoptic and height maps, as well as the NCEP/DOE AMIP II reanalysis data, which made it possible to investigate the trajectories of motion of baric formations and calculate the amount of formations of various geneses.

Dynamic processes affecting the existence, intensity, and duration of atmospheric blocking regimes (blockings) are permanently in the field of vision of researchers. On the whole, the blockings are believed to be the results of an interaction between intensifying synoptic-scale waves transferring the anticyclonic vorticity into the region of blocking and a quasi-stationary planetary-scale wave. When the variability of atmospheric circulation is analyzed, short-period (with time scales of 2–6 days) and low-frequency (with a duration of more than 10 days) processes are identified. The maximums of low-frequency variability are located in regions with the maximal recurrence of blocking situations [38]. The lifecycle of atmospheric blockings and, in particular, the stage of their destruction, are of special interest. Lupo et al. [39] analyzed three blocking events which occurred in the Southern Hemisphere over the Pacific Ocean in cold

seasons. An analysis of phase trajectories showed that abrupt changes in the large-scale structure of atmospheric flows can lead to a rapid decay of a blocking. The cited paper proposes four different scenarios of such decay: at a weakening of synoptic “replenishment,” at a high activity of synoptic processes, and each of these regimes in interaction with sharp changes in the planetary flow character.

The progress achieved in recent decades in the development of atmospheric general circulation models, as well as in technologies of assimilation of satellite observational data, is responsible for the interest of researchers in the problem of the reproduction and prediction of seasonal climate anomalies. The World Meteorological Organization, within the framework of the World Program on the Climate Study, initiated projects devoted to a comparison of atmospheric models in relation to the reproduction of seasonal climatic anomalies. Tolstykh et al. [40] presented the results of reproduction of the atmospheric circulation on seasonal time scales with the use of the global semi-Lagrangian model developed at the INM RAS and Hydrometeorological Center of Russia. With the aid of this model, the authors of the cited work calculated ensembles of retrospective seasonal predictions (over 25 years for each season) on the basis of the NCEP/NCAR reanalysis data within the protocol of the international SMIP-2/HFP experiment aimed at estimating practical predictability on seasonal time scales. A comparative analysis of the natural orthogonal components calculated for the main seasons from the model data and the reanalysis data for the geopotential of the surface 500 hPa and sea-level pressure showed that they agree quite satisfactorily.

Klimova et al. [41] described a method for estimating the statistical structure of errors in the short-term prediction of the temperature field in the atmospheric boundary layer for the purposes of objective analysis. Numerical experiments of the estimation of covariances in prediction errors were conducted for the WRF (Weather Research and Forecast, NCEP, United States) model. The cited work describes the results of numerical experiments on the estimation of the discussed covariances in the atmospheric boundary layer depending on its stability in the summer and winter periods. It is shown that the variance and vertical radius of the correlation of prediction errors in the temperature field, as well as the behavior of three-dimensional covariance functions in the atmospheric boundary layer, differ substantially at different characters of stability.

The approach based on variational principles with the use of methods of assimilation and a combination of the main and adjoint problems for models of processes is effective for organizing the interaction between prognostic models and data. To date, two directions have been developed. The first direction includes methods of optimization originating from the method of weighted least squares (the Sasaki method

and the Kalman filter). The second direction is based on the classical Lagrange variational principle with the use of adjoint problems.

The Kalman filter (KF) is known to be an optimal method of successive data assimilation for the linear dynamics. However, in the case of nonlinear models of a large dimensionality, the KF encounters serious difficulties. First, if a dynamic model has the vector of state of the dimensionality  $N$ , the matrix of error covariances will have the dimensionality  $N^2$ , which calls for the storage of large data arrays and a large volume of computations. Second, the linearization for obtaining the equation of evolution of the covariance of errors is necessary for the use of the KF with a nonlinear dynamics. This requirement introduces errors in the prediction of covariances, which can grow in the case of instability. The use of the high-order closure schemes makes this method practically inapplicable for the problem of data assimilation. Therefore, another method of successive data assimilation, namely, the ensemble Kalman filter (EKF), has become popular in recent times. It removes the limitations of the determinate KF.

Klimova [42, 43] proposed methods for observational data assimilation based on the EKF ideas under the assumption that random prediction errors possess the property of ergodicity. In this case, the algorithm, which changes the probability averaging to the averaging over time ( $\pi$  algorithm), is obtained. The author of the cited work investigated the applicability of the  $\pi$  algorithm in the data assimilation problem using a simple one-dimensional equation of advection as an example. The use of such a simple equation made it possible to compare this algorithm with the classical algorithm of the Kalman filter and to consider different approaches to its practical realization.

Penenko [44] proposed new methods of variational data assimilation and formulated inverse problems for the identification of model parameters. The explicit inclusion of uncertainties calls for Tikhonov regularization. For these purposes, functionals expressing the summarized measure of uncertainties are introduced into the formulation of variational principles. The author of the cited work noted that, at such an organization of data assimilation methods, feedbacks from data to models are switched on in the entire interval of assimilation. This is a new basic element of the method, because, in traditional assimilation methods, the feedback is usually switched on at the initial time of the assimilation “window.” Methods of successive data assimilation in the real time are of great interest for practical applications. Here, the assimilation window is equal to the step of model discretization in time and the algorithm takes place without iterations. Additive—averaged schemes of splitting and discrete—analytical monotonical schemes for convective—diffusive operators are used for the discretization of models and functionals. Algorithms with the parallel organi-

zation of calculations are proposed for the fulfillment of these schemes.

### 3. MESOSCALE PROCESSES

Polar mesoscale cyclonic vortices forming in the cold season over the sea surface free of ice are a bright feature of the atmospheric circulation at high latitudes. The characteristic size of these vortices varies from several tens to several hundreds of kilometers. Mokhov et al. [45] analyzed polar mesocyclones by using the data from the archive of images of 253 vortical formations detected over the water area of the North European basin, Barents Sea, and Kara Sea in the period from 1981 through 1995. In spite of the considerable interannual variations in the parameters of these cyclones, no substantial climatic trends were revealed. It also turned out that the cumulative distribution of the recurrence of Arctic mesocyclones over the region under consideration can be approximated sufficiently well by the exponential function in the size range from 50 to 400 km.

At present, a rather large amount of attention is being focused on the problem of tropical cyclones in connection with the observed climate changes: will their number and intensity increase in a warmer climate? It is intriguing that tropical cyclones occur only at an ocean surface temperature  $T_s$  no lower than 26°C. Golitsyn [46, 47] considered tropical cyclones and polar mesocyclones with an “explosive” (for several hours) development as unified hydrodynamic structures in the form of intense vortices which appear over the ocean in the atmosphere of a rotating planet. The ocean upper layer is an energy source for such vortices, and the atmosphere must be colder than the water surface and not saturated in moisture up to 100%. In the polar region, the ocean gives up its heat as a sensible heat flux, whereas in the tropics the latent heat flux plays the main role. In the cited papers, G.S. Golitsyn used considerations of the similarity and dimensionality theory in order to estimate buoyancy fluxes, as well as the fluxes of sensible and latent heat separately on the basis of climatologic data, aerodynamic formulas for the near-water layer, and the convection rate scale in a rotating fluid. It turned out that, in the tropics, at hurricane-strength winds with  $U \geq 33$  m/s and the climatologic air humidity 80%, the total heat flux at the water surface temperature  $T_s \geq 26^\circ\text{C}$  becomes close to 700 W/m<sup>2</sup> and even can exceed this value. Owing to the Clausius–Clapeyron equation, the latent heat flux into the atmosphere decreases substantially at smaller  $T_s$  values. For the penetration of convection above the boundary layer, a substantially weakened static stability of the atmosphere and the absence of noticeable wind shears in height are necessary, along with intense buoyancy fluxes. For the formation of explosive mesocyclones, the total heat fluxes in the polar region must be approximately twice as large as in the tropics due to the substantially smaller

role of latent heat, larger geostrophicity, and the larger stability of the atmosphere.

According to present-day notions, the instability of barotropic Rossby waves is one of the possible mechanisms of formation of regular and quasi-regular vertical chains of a synoptic scale (in particular, circumpolar cyclones and anticyclones). It is shown in work [48] that, depending on the parameters of the circumpolar jet flow, a periodic, quasi-periodic, or chaotic regime of generation of planetary waves is established in the process of barotropic instability occurring. In this case, chaotic advection and the diffusion of passive admixture particles in chains of vortices are an important problem.

Shagalov et al. [49] investigated the generation of spectrally narrow Rossby wave packets and vortical chains produced by them in a zonal flow with a shear velocity profile. The asymptotic approach used for the analysis was based on the identification of a narrow critical layer in which vortical chains are formed. It is shown that the supercriticality increase during the secondary instability development leads, first, to Lagrangian chaos (a chaotic motion of particles) and then (with its further increase) to the chaotization of the vortical field. The authors of the cited work also investigated the motion of passive admixture particles in conditions of stationary self-consistent regimes of the generation of Rossby waves and chaotically modulated vortical chains accompanying them. It is shown in the cited work that the so-called anomalous diffusion, which, compared with the ordinary diffusion, leads to a more effective spreading of admixtures along chains of barotropic vortices, appears during the increase in supercriticality.

The substantial contribution of ordered (convective) flows to the total vertical transport is noted in numerous experimental investigations of the admixture diffusion in a turbulent atmosphere (see, for example, the review in paper [50]). Kukharets et al. [51] theoretically investigated the vertical transport of a passive admixture in a medium in the presence of both turbulent and convective mixing. It is shown that the turbulent and convective transport mechanisms mutually affect each other. The effect of this influence manifests itself, in particular, in the fact that the presence of convection weakens the turbulent transport, which is caused by the “trapping” of admixture particles by vortical structures, and leads to its anisotropy even in an isotropic turbulent velocity field.

Gorbatenko and Konstantiniva [52] and Konstantinova and Gorbatenko [53] investigated the specific features of convection over Siberia in connection with dangerous weather phenomena. The main goal of such investigations consists in the elaboration of a method for estimating the probability of the development of mesoscale convection to dangerous weather phenomena. The cited publications analyze the spatial and temporal variabilities of the parameters characterizing convection while invoking a number of indicators:



temperature stratification, moisture content in lower layers of the atmosphere, its energy potential, and abrupt changes in wind characteristics with height. For a study of convection and dangerous weather phenomena caused by its development, the authors of the cited works used data on the state of the atmosphere obtained with the aid of the upper-air sounding of the atmosphere. For the study of convective cloudiness, Ananova et al. [54] considered, along with traditional observational data, the radar characteristics of cloudiness during a squall in southwestern Siberia. The recurrence of the maximum height of cumulus–rain cloudiness in the presence of a squall, the zero isotherm height, the radar reflectivity at three levels, and the maximum reflectivity, as well as the complex criterion of thunderstorm hazard, were investigated. The radar characteristics of cloudiness were determined during squalls in various synoptic situations.

Forest fires, which have substantial influence on the gaseous and aerosol compositions of the atmosphere, are responsible for noticeable regional changes in the environment and can adversely affect human health. In addition, large amounts of aerosols ejected into the atmosphere favor cloudiness, thereby affecting circulation processes. For an investigation into atmospheric circulation during forest fires, Aloyan [55] used a regional hydrodynamic model based on the joint solution of the problems of atmospheric dynamics in a humid atmosphere, as well as the kinetic processes of condensation and coagulation. The cited paper pays particular attention to a description of convective processes with consideration for heat fluxes from the zone of burning. It is shown that the release of condensation heat increases the height to which soot particles rise.

The influence of a megacity on the temperature regime of the atmospheric boundary layer manifests itself, in particular, in the formation of a so-called “heat island” over a city, the existence of which is supported by data from measurements on meteorological masts and television towers, as well as with the aid of balloon radio sounding at suburban aerological stations. Kadygrov et al. [56] analyzed the data of temperature measurements conducted with a microwave radiometer at three sites, one of which is located in the center of Moscow (megacity) and two others are outside Moscow (in the town of Dolgoprudnyi (suburb) and at the IAP RAS Zvenigorod Scientific Station (background)). An analysis of measurement data showed that disturbances introduced by a megacity into the diurnal and seasonal variability of the temperature in the atmospheric boundary layer dampen with height but remain statistically significant up to a height of 600 m.

The main feature of the problem of the air quality over the urbanized inhomogeneous surface is a wide spectrum of spatial–temporal scales of the processes controlling this quality, among which we can identify the scale of the city (tens of kilometers) where the pri-

mary emission of air pollutants takes place, as well as the micro- and mesoscales on which the secondary air pollutants are formed and dispersed. The dispersion of pollutants depends heavily on the structure of the atmospheric boundary layer and its interaction with the background flow and underlying surface. Models of turbulence with a high spatial resolution, which take into account the chemical transformations of pollutants, are required for the numerical modeling of such a system.

Due to the increase in traffic intensity and the organization of small production plants with uncontrollable emissions of hydrocarbons, the problem of atmospheric pollution from formaldehyde has become especially important for most Russian cities. Shlychkov et al. [57] attempted to reconstruct the field of formaldehyde concentrations in the city of Tomsk with the use of a numerical model and observational data. The wind regime over the orographically inhomogeneous territory of Tomsk and its vicinities was determined on the basis of diagnostic calculations with the use of a one-layer mesoscale model, whose basic equations were obtained from compositions of statistical and hydrodynamic methods and on the basis of the hypothesis of the “vertical” similarity of hydrodynamic fields in the surface layer. The three-dimensional equation of transport and diffusion of a passive admixture was used for calculating the formaldehyde distribution over the territory of the city. The calibration of parameters ensured the adequacy of the calculated characteristics to the data of formaldehyde concentration measurements at observational posts. The model can be used for obtaining relative estimates of the contributions of individual plants or source groups (including motor transport emissions) to the total pollution of the urban atmosphere, as well as a component of expert systems or the systems supporting the adoption of administrative solutions concerning, in particular, the development of preventive measures favoring the improvement of the air quality of the city.

Penenko et al. [58] described a complex of models of mesoscale transport of admixtures, including the determinate model in the Eulerian formulation and the determinate–stochastic model within the framework of the Lagrangian approach. The cited work presents the results of comparative experiments on the modeling of admixture transport in regions with a complex geometry.

The results of complex ship-board investigations of the spatial distribution of aerosol fields carried out in Lake Baikal with the use of laser sounding and local control are presented in the cycle of works [59–61]. It is detected that the mean total concentrations of polycyclic aromatic hydrocarbons (PAH) over the Baikal water area are close to the background ones, and their spatial distribution is very inhomogeneous. The joint analysis with two-dimensional spatial lidar sections of aerosol fields revealed characteristic tendencies in spatial fluctuations of these quantities. In addition to

the instrumental investigations, the authors of the cited works performed model calculations of the PAH distribution based on the numerical solution of the spatial nonstationary semi-empirical equation of turbulent diffusion of admixtures. The comparison with the data of the experiment showed that the calculated characteristics are basically close to the measured values. The concentrations of nitrogen oxides measured over the Baikal water areas in the summer of 2005 are analyzed in comparison with the meteorological conditions over the same period. The regions of increased pollution by nitrogen compounds in the Lake Baikal area are revealed with the use of the numerical model of propagation and transformation of admixtures. The situation associated with the propagation of the smoke train from forest fires on the northwestern coast of the lake is analyzed.

Kurbatsky [62] showed that the nonlocality of the mechanism of turbulent heat transfer in a mesoscale boundary layer over a rough surface manifests itself in the form of restricted zones of countergradient heat transfer. These zones are identified through the analysis of balance items in the equation for the dispersion of temperature fluctuations based on the calculation of the coefficients of turbulent exchange of momentum and heat with invoking the model of “gradient diffusion.” In this case, the countergradient heat transfer in local regions is caused by turbulent diffusion or the term of divergence of triple correlation in the equation of the temperature dispersion balance.

Recently, more and more attention is being paid to the investigation of processes of heat and mass exchange between the atmosphere and the landscape-inhomogeneous land with different types of interface between these two media. The traditionally used Monin–Oboukhov parametrizations in many cases do not yield satisfactory agreement with the data of full-scale experiments. Panin and Bernhofer [63] analyzed the data from a measurement of the heat balance components over various land surfaces, and the results of this analysis indicate that the sum of fluxes of sensible and latent heat is systematically smaller than the difference between the radiation balance and the heat flux into the soil; this unbalance increases with the growing inhomogeneity of the landscape. This fact can point to the important role of inner boundary layers and atmospheric circulations associated with them in the parametrization of subgrid effects in large-scale models. In the cited paper, empirical formulas are constructed for the correction of turbulent heat fluxes over natural (inhomogeneous) land surfaces measured or calculated on the basis of the aerodynamic method.

#### 4. TURBULENCE IN THE BOUNDARY LAYER

A wide spatial–temporal spectrum of turbulent pulsations, of which the longest wave and lowest frequency ones can attain several kilometers and have time scales of several hours, is one of the main features

of turbulent flows in the atmospheric boundary layer (ABL). It is also important that the formation of long-lived large-scale quasi-ordered structures (large thermics, whose shape is close to hexagonal Benard cells; convective rolls; and helical extended vortices of various origins), which control a considerable part of the integral transfer of momentum, heat, and moisture, is characteristic of the boundary layer. In the turbulent flow, the mean flow and velocity fluctuations in the entire range of scales continuously interact. In this case, as a rule, even in the presence of quasi-ordered structures, the flow cannot be uniquely separated into small-scale and large-scale components, because the energy spectrum of velocity fluctuations in turbulent boundary layers does not have a clearly pronounced minimum.

The eddy-resolving modeling (Large Eddy Simulation (LES) in the English language literature) is a powerful tool for investigating the nonstationary three-dimensional dynamics of large-scale vertical structures in shear flows (including density-stratified ones) at very large Reynolds numbers  $Re \gg 1$  (in particular, on the order of  $10^9$  for the atmospheric boundary layer). A consistent choice of the parametrization of small-scale turbulence and the numerical scheme is an important aspect of the development of eddy-resolving models. Such a model was presented by Glazunov [64, 65]. This model is based on the spatially filtered Navier–Stokes equations, a mixed localized turbulent closure of the dynamic type, and a conservative scheme of the fourth order of accuracy. A substantial part of nonlinear interactions in the region of large (but still presentable on the model grid) wave numbers is replaced by a dissipative term, which is justified if the direct energy cascade prevails in a three-dimensional turbulent flow. The dynamic turbulent closure automatically controls the sink of the kinetic energy of small-scale vortices so that this model not only reproduces the structure and energy of large-scale components of the turbulent flow but also makes it possible to estimate the spectral density of the energy of velocity pulsations in the region of large wave numbers.

However, it was detected that, in LES models with explicit filtering, the reverse energy cascade predicted by the dynamic closure is controlled by interactions of permitted pulsations with close in scale “subfilter” harmonics. At the same time, the reversibility of the filtering operator makes it possible to obtain from the prognostic velocity its “reconstructed” analog, which more exactly reflects the small-scale structure of the flow, for any moment in time [65]. In this work the model was verified with the aid of a number of prolonged numerical experiments on the reproduction of turbulent flows in a channel bounded along the vertical by two identical infinitely extended rough plates. The motion of a fluid in the channel was sustained through an external force directed along its axis, which was specified by a constant pressure gradient. The results of calculations were compared with the data of

laboratory experiments and the results of direct numerical modeling (DNS). It was established that a priori reconstruction of the modeled velocity field ensures a substantially more exact reproduction of statistical characteristics of the model solution.

The influence of rotation on the intensity and structure of turbulent pulsations was established rather long ago (see, for example the experimental data confirming the turbulence suppression or enhancement in a shear boundary layer depending on the rotation direction in paper [66]). The authors of work [67] performed a series of calculations with the use of the LES model of the neutrally stratified ocean upper layer at different directions of the friction stress vector on the surface. The results of these calculations showed a strong dependence of the intensity of turbulent processes on the flow direction. With the use of the eddy-resolving model of the atmospheric boundary layer, Glazunov [68] studied the influence of the Earth's rotation on the structure of turbulence and dynamics of quasi-ordered vortices. The numerical experiments for the neutrally stratified turbulent Ekman layer with large dimensions of the computational region (21 km in both horizontal directions and 3 km along the vertical) and with a grid size of  $\sim 40$  m (about 20 million nodes of the finite-difference grid) made it possible to simultaneously explicitly reproduce small-scale three-dimensional turbulence and large-scale rolls several kilometers in size. It was established that the presence of the meridional component of the angular velocity of the Earth's rotation significantly enhances the intensity of velocity fluctuations in the neutrally stratified turbulent flow at easterly and northeasterly winds and weakens the intensity of fluctuations at westerly and southwesterly winds. These circumstances, in turn, cause significant changes in the mean velocity profile. It is shown that these changes are associated with the largest scale fluctuations, which are comparable in scale with the thickness of the turbulent Ekman layer. It is detected that, in the limited-in-height neutrally stratified atmospheric boundary layer and at its stable stratification, the dependence on the wind direction substantially decreases.

With the aid of the eddy-resolving model, Glazunov et al. [4] investigated (from the standpoint of reproducing spectral properties) the Rayleigh–Benard thermal convection in a doubly periodic channel with hard walls as an analog of multiscale atmospheric turbulence. The mesoscale ratio of the horizontal dimensions of this channel to its vertical dimension (25.6 in both directions) ensured the existence of quasi-two-dimensional large-scale components of the flow, and the size of the regular computational grid (about 42 million nodes) made it possible to explicitly reproduce the dynamics of the small-scale three-dimensional turbulent component. An analysis of the results of numerical experiments showed that convection begins with small-scale chaotically located upwardly floating and descending thermics, which, uniting,

form convective cells of an irregular shape (including deformed hexagonal cells) of approximately the same size as the distance between the walls. Then these cells start to merge with each other and enlarge until the size of the largest anomalies attains the size of the computational region of the model. At each moment in time, small-scale anomalies are observed against the background of large cells.

Decomposition of the studied turbulent flow into the barotropic and baroclinic components made it possible to propose the following scheme of conversions of the kinetic energy in this system. The kinetic energy is supplied into the system owing to the conversion of the available potential energy into the baroclinic kinetic energy on the scale of large thermics (through the vertical velocity component) and is redistributed on the same scale through pressure gradients into baroclinic components controlled by the horizontal velocity components. Due to nonlinear interactions and without substantial dissipation and generation, the baroclinic energy is transferred toward small scales, forming the first inertial interval with the spectral distribution close to the law  $k^{-5/3}$  ( $k$  is the wave number). In the interval of wave numbers associated with scales close to the vertical dimension of the computational region, the field of baroclinic velocity fluctuations undergoes a substantial reorganization, which ensures the energy conversions from barotropic into baroclinic and vice versa with the positive, on average, contribution to the energy of flows averaged over the entire layer thickness. The energy of the barotropic component propagates from its source mainly toward large scales, forming the spectral dependence of the form  $k^{-5/3}$ , as well as, to a lesser degree, toward small scales, which, as a result of the enstrophy cascade, leads to the distribution  $k^{-3}$ . The remaining baroclinic kinetic energy, which was not converted into the barotropic component, is transferred through the direct cascade of nonlinear interactions toward small scales, where its dissipation takes place (in the case of an eddy-resolving model due to the dissipative contribution of the closure and in the case of a real turbulent flow due to the forces of molecular viscosity).

Gertsenshtein et al. [69] considered in the Boussinesq approximation two-dimensional convective flows of a viscous incompressible fluid in an infinitely extended layer between two horizontal plates heated from below. With this purpose, the direct numerical modeling of these flows was carried out by the Bubnov–Galerkin method within the framework of the nonstationary Navier–Stokes equations. The problem is considered in two formulations differing in boundary conditions at horizontal boundaries. In the case of the so-called “free” conditions, it is assumed that the vertical velocity, tangent friction stress, and temperature vanish at the horizontal boundaries. In the formulation with the “rigid” conditions, no-slip conditions are used instead of the vanishing of the tangent friction stress. The calculations are performed for different

values of the supercriticality parameter  $r = Ra/Ra_{cr}$ , where  $Ra = g\beta H^3 \delta T / \nu \chi$  is the Rayleigh number ( $H$  is the layer thickness;  $\delta T$  is the difference of temperatures at the boundaries of this layer; and  $\beta$ ,  $\nu$  and  $\chi$  are the coefficients of thermal expansion, kinematic viscosity, and thermal diffusivity) and  $Ra_{cr}$  is the critical value of the Rayleigh number depending on the type of the problem (equal to 657.5 and 1708, respectively). Particular attention in the cited work is given to the spatial spectra calculated for the cases of a high supercriticality (at  $r = 26000$  in the problem with the free boundary conditions) and a relatively low supercriticality (at  $r = 6000$  in the problem with rigid conditions). In the region of small wave numbers, the spectra are close to the Kolmogorov spectrum  $k^{-5/3}$  and distribution close to the law  $k^{-1}$  is formed in the short-wave branch of the spectral curve. In this region, the spectral law  $k^{-3}$  is fulfilled for the kinetic energy of pulsations. Such a “coexistence” of the power laws  $k^{-1}$  and  $k^{-3}$  points to the presence of the inertial interval of the direct enstrophy cascade.

Processes of the turbulent interaction of the atmosphere with the ocean surface are critically important in the theory of hurricanes and polar mesocyclones. The coefficient of aerodynamic drag of the sea surface  $C_d$  is one of the characteristics of such an interaction. The traditional aerodynamic formula based on the generalization of experimental data for wind velocities smaller than 30 m/s yield overestimated  $C_d$  values at hurricane-strength winds. On the basis of a generalization of the results of measurements with the aid of GPS sondes dropped inside tropical cyclones, it was shown in paper [70] that the drag coefficient decreases if the velocity of near-water winds attains 30–35 m/s. This effect may be caused both by changes in the sea surface configuration in energy-carrying waves, which are accompanied by the appearance of a sharp fore-front and the detachment of the atmospheric boundary layer [71], and by the mechanism associated with the presence in the flow of sprays, which formed as a result of the breaking away of steep wave crests by the wind [72, 73].

Troitskaya and Rybushkina [74] proposed determining the aerodynamic coefficient of the drag of the ocean surface at hurricane-strength winds with the use of a quasi-linear model of the wind boundary layer based on the solution of the Reynolds equations with allowance for the effects of a viscous sublayer. Within the framework of this model, the effect of the  $C_d$  decrease at hurricane-strength winds is attributed to the fact that the wind generation of waves causes the transfer of momentum from wind to waves, which strongly decreases the turbulent stress near the surface. This leads to the eddy viscosity decrease near the surface and deforms the wind velocity profile. A comparative analysis of the results of calculations and the experimental data for a wide range of wind velocities made it possible for the authors of the cited paper to

propose a simple parametrization of the drag coefficient for use in numerical models of wind and wave predictions.

The land surface is one of the sources of aerosol in the atmosphere. At certain conditions, soil particles from the land surface can be carried out into the atmosphere (for example, during dust storms and dust whirlwinds). Experimental data and theoretical estimates indicate that the detachment of particles from the soil surface can be caused by turbulent friction stresses in the conditions, when the dynamic velocity exceeds a certain critical value (see, for example, [75]). Gledzer et al. [76], on the basis of the data of full-scale measurements in the Caspian desert and the estimates of hydrodynamic parameters in the viscous thermal layer adjacent to the soil surface, obtained asymptotic expressions for the mass concentration of finely dispersed aerosol. It is supposed that the exportation of aerosol from the soil is proportional to the air velocity at the level of the thermal layer, which is controlled by the dynamic velocity and temperature deficit in this layer. The model of the air dynamics in a porous air layer with the invocation of the Darcy law is considered in the cited work as a possible mechanism of aerosol exportation.

Kurbatsky and Kurbatskaya [77, 78] presented the results of investigating specific features of the structure of a stable boundary layer over an urbanized surface. With this purpose, the authors of the cited works developed a modified three-parameter model of turbulence for a thermally stratified atmospheric boundary layer. This model is based on tensor-invariant parametrizations for the pressure stress and the correlation between pressure and temperature. The turbulent fluxes of momentum and heat are calculated with the aid of algebraic explicit models identically describing the state of convective mixing, a stably stratified regime, and transitions between them. A comparison of the results of modeling with the observational data and the results of other models showed that the proposed model is capable of reproducing the most important features of the turbulence structure in the boundary layer over the urbanized surface, as well as the effect of its roughness on the global structure of the wind and temperature fields over a city. At present, it is generally accepted that, in stably stratified flows of the atmospheric boundary layer, the mixing exists at the gradient Richardson numbers  $Ri_g \gg 1$  and the inverse turbulent Prandtl number  $Pr_t^{-1}$  decreases with the increasing thermal stability of a flow. It is shown in paper [79] that the abovementioned three-parameter model of turbulence, which takes into account the effect of stratification in the expression for the time scale of a scalar field, can reproduce the dependence of the Prandtl number on the Richardson number (known from experimental data and the results of LES modeling), as well as the countergradient heat transfer in a strongly stable boundary layer.

## 5. MATHEMATICAL PROBLEMS OF CLIMATE AND ECOLOGY

The direction associated with the development of the mathematical theory of climate based on the use of methods of the theory of dynamic systems intensely gathered force in 2007–2010. The elaboration of the methodology for estimating the sensitivity of a climatic system to small external forcing, which would provide a constructive method for calculating climate changes under the influence of this forcing, is one of the main purposes of such a theory. In this case, the notion “climate” is introduced as a set of states passed by a climatic system over a fairly long time interval (in practice, this interval is assumed to be 30 years and longer), and the problem of global and regional climate changes is interpreted as a problem of the sensitivity of statistical characteristics of solutions to the system of equations describing the dynamics of a real climatic system. Mathematically, the problem of climate sensitivity is a problem about the sensitivity of the climatic system attractor (a set of states in the phase space where the evolution of the system takes place) and its invariant measure (the equilibrium distribution of system states on the attractor) to changes in the system parameters [38, 80]. It is assumed in the study of the response of the climatic system to small external forcing that the dynamics of a climatic system takes place on its attractor, and models, which in some or other manner successfully describe the present-day climate, can be used for its qualitative analysis. However, it is important to bear in mind that typical models of the climatic system possess the property of a chaotic state caused by the presence of positive Lyapunov exponents and are dissipative, so that their attractors have a fractal topological structure.

The construction of an approximate response operator on the basis of fluctuation–dissipation relations (FDR), which relate the response operators of statistical characteristics of a model to its undisturbed statistical characteristics, is a promising method for estimating the sensitivity of climatic models and a real climatic system to external forcing [81]. This means that the response operator of a system can be calculated solely from the data of an undisturbed trajectory of a system and, consequently, the sensitivity of some characteristics of a real climatic system to changes in the external parameters can also be calculated directly from observational data. The FDR technology of constructing the response operator requires a calculation of multidimensional covariance matrices of the system and their subsequent inversion, which represents a complex computational problem. Gritsun and Branstator [82] demonstrated in what way this technology can be used for constructing the linear part of the response operator on the basis of data obtained with the aid of a (nonlinear) model of atmospheric circulation. The CCM0 model was used for this purpose. This model was developed at the National Center of Atmospheric Research (NCAR, Boulder, United

States) and was integrated in the “continuous January” regime for 4 million days. The patterns of stationary responses in the temperature and stream function fields to disturbances in heat sources calculated with the use of the FDR technologies and directly (with the use of the CCM0 model) showed a rather good agreement between them. It was also found that it is possible to formulate and solve the inverse problem, i.e., to find the disturbance which will be optimal for the specified response. Gritsun et al. [83] generalized the FDR method for constructing the response operator to the case of climatic characteristics, which are not parameters of the system state but represent their certain functionals, such as the averaged precipitation, the mean divergence in the upper troposphere, or the low-frequency variability of the stream function. In particular, the optimal tropical disturbances responsible for maximal changes in the variability of synoptic vortices at midlatitudes are constructed. However, it should be noted that the main limitation of the FDR method is a requirement for the statistical equilibrium of an undisturbed system, which is impossible if the right-hand side of the system contains the explicit time dependence. The recently obtained generalized FDRs [84] offer the theoretical possibility for constructing the response operator in this case as well.

Evidently, at present, the only method of investigating attractors of multidimensional climatic systems is the numerical approximation of corresponding sets [85], for example, the calculation of global Lyapunov exponents (as indicators of the measure of instability of the system trajectory on the attractor) and the system attractor dimensionality (as a measure of the system dynamics complexity). The method of describing the fractal attractor of the system through its approximation with the aid of simple basic sets, for example, periodic orbits, seems promising. The solution to problems related to the existence of periodic solutions in the phase space of atmospheric models and the establishment of relations between the characteristic regimes of circulation and the properties of the model phase space, as well as the elucidation of the possibility to approximate circulation with the aid of periodic motions, are given by Gritsun in the cycle of works [86–89].

In the first work [86] of this cycle, several methods of the search for periodical orbits of the model are formulated and achieved for the barotropic model of the atmosphere. These methods made it possible to find more than 1500 periodic solutions to the model. The calculated orbits have a wide range of periods (from 4 to 200 days) and different characteristics of stability (from 2 to 30 unstable modes). The next paper [87] considers the problem of approximating the invariant measure and statistical characteristics of the barotropic atmosphere model with the aid of its periodic trajectories. The possibility of such an approximation is based on ideas of the theory of dynamic systems stating that, in some particular cases (for example, for

hyperbolic systems), periodic trajectories control the measure of the system. In this case, orbits are taken into account in accordance with the weight controlled by the characteristics of their instability. It is shown in the cited paper that, using an appropriate method of averaging, it becomes possible to reproduce the probability density of the distribution of points in the phase space of the system with an error not exceeding 10%. In conclusive papers [88, 89] of the cycle mentioned above, the method of generalized FDRs was used for constructing an approximate response operator of the CAM3 NCAR model. With this purpose, the author of the cited works proposed the numerical method for constructing the response operator and preliminarily calculated the model response to equatorial thermal actions.

Investigations into improving and developing methods for solving ecological problems under the conditions of changes in the climate and the quality of environment were carried out in 2007–2010 [90–92]. The specificity of this class of problems consists of the necessity of considering a wide spectrum of interacting processes proceeding in long time intervals in external and internal sources of disturbances. It is also necessary to take into account feedbacks when changes in the climatic system are caused by anthropogenic and natural impacts. Relations between the sensitivities of models and functionals of generalized estimates provide the constructive basis for the formation of forward and backward links between different elements of the system of modeling. The functions of sensitivity emerging in this case synthesize the solution of forward, adjoint, and inverse problems for mathematical models of hydrodynamics, transport, and transformation of admixtures, as well as the calculation of relations of the sensitivity of the atmosphere quality functionals. Penenko and Tsvetova [90, 91] proposed a method of ecological prediction with allowance for changes in climatic factors. A set of subspaces ranging in scales of disturbances is identified with the aid of the orthogonal decomposition from multidimensional multicomponent databases containing information about the functions of state describing atmospheric processes over a long period. The leading part of subspaces taking into account processes of the climatic scale is an informative basis for the formation of the hydrodynamic background for the calculation of prognostic scenarios of changes in the quality of the atmosphere. The method is elaborated with the use of the NCEP/NCAR reanalysis database over more than 50 years. The cited works present the results of scenario calculations for estimating the risk of atmosphere pollution in the Far East region of Russia and the adjacent territories of China and Korea. Penenko [92] describes the concept of environmental prediction based on variational principles combined with methods of observational data assimilation, as well as on the estimation of risk and vulnerability of territories with respect to anthropogenic impacts, which makes it

possible to estimate the influence of uncertainties on the prediction quality.

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